



# Interactions of fluid oxygen drops in fluid hydrogen at rocket chamber pressures

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## Abstract

A model of fluid drop behavior in clusters has been developed including the interactions induced by the drop proximity. The model is based upon the global conservation equations for the interstitial cluster region coupled to isolated fluid drop equations previously developed. Heat and mass transfer to the cluster are modeled using the Nusselt number concept. Results from calculations for the  $\text{LO}_x\text{-H}_2$  system show the predictions to be insensitive to the value of the Nusselt number over three orders of magnitude. The results also show that at fixed pressure, increased drop proximity induces increased accumulation of  $\text{LO}_x$  in the interstitial space inside the cluster. At fixed initial drop proximity, the gradients of the dependent variables become increasingly smeared as the pressure increases; an opposite result from that obtained for isolated drops (Harstad K, Bellan J. Isolated fluid oxygen drop behavior in fluid hydrogen at rocket chamber pressure. *Int J Heat Mass Transfer* 1998;41:3537–50). It is thus inferred that clusters of drops might be a desirable aspect in supercritical combustion because they aid interdiffusion of the reactive components. © 1998 Elsevier Science Ltd. All rights reserved.

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## Nomenclature

$A$  area of a surface  
 $A_j, A_q$  coefficients in the transport matrix  
 $B_j$  coefficient in the transport matrix  
 $C_p$  molar heat capacity at constant pressure  
 $C_q$  coefficient in the transport matrix  
 $D$  diffusion coefficients  
 $h$  molar enthalpy  
 $J$  molar flux  
 $Le$  Lewis number  
 $Le_{\text{eff}}$  effective Lewis number  
 $m$  molar mass  
 $M$  total mass  
 $n$  number of moles per unit volume  
 $N$  number of drops  
 $N_d$  drop number density in cluster  
 $Nu_C$  equivalent Nusselt number  
 $p$  pressure  
 $q$  heat flux

$r$  radial coordinate  
 $R_{si}$  radius of the sphere of influence  
 $R_d$  drop radius  
 $R_u$  universal gas constant  
 $S$  surface of a defined volume  
 $t$  time  
 $T$  temperature  
 $v$  molar volume  
 $V$  volume  
 $x$  generic coordinate  
 $X$  mole fraction  
 $Y$  mass fraction.

## Greek symbols

$\alpha_D$  mass diffusion factors  
 $\alpha_T$  thermal diffusion factor  
 $\alpha_v$  thermal expansion ratio  
 $\kappa_s$  isentropic compressibility  
 $\lambda$  thermal conductivity  
 $\rho$  density.

## Subscripts

b boundary  
C cluster

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d drop  
 e external to cluster  
 i interstitial  
 j species  
 m mass  
 si at the edge of the sphere of influence  
 T heat.

#### Superscripts

i interstitial  
 0 initial value.

## 1. Introduction

In liquid rocket combustors, atomization of the liquid oxygen occurs either by means of coaxial jets of  $\text{LO}_x$  surrounded by a hydrogen stream, or by means of impinging jets (either  $\text{LO}_x$ – $\text{LO}_x$ , or  $\text{LO}_x$ –fuel which promotes mixing of reactants). Experimental observations of atomization of coaxial jets by Hardalupas et al. [1] and Engelbert et al. [2] reveal the initial formation of ligaments, each ligament quickly disintegrating into a cluster of drops. Similarly, visualizations of impinging liquid jets [3] have shown that under both cold and hot flow conditions the jets break up in a periodic manner into ligaments which further break up into drops thereby creating clusters of drops. Recent observations by Ryan et al. [4] have documented this typical breakup process while also yielding information regarding the breakup length and the drop size of both laminar and turbulent impinging jets. Poulikakos [5] made similar observations.

These observations indicate that fluid drops created during atomization in liquid rocket chambers do not behave as isolated and instead have a collective behavior. This realization is very important for controlling high frequency combustion instabilities because the response function must incorporate this fluid drop interaction.

Fluid drop interaction at high pressure has been modeled by Jiang and Chiang [6] using the concept of ‘sphere of influence’ of Bellan and Cuffel [7] and results were obtained for *n*-pentane adiabatic drop arrays where the drops are uniformly distributed and stationary. The model of Jiang and Chiang [6] does not include the intricacies of high pressure transport processes and thus it is not appropriate for the supercritical regime where Fick’s and Fourier’s laws are no longer solely valid to completely describe the transport matrix [8]. Additionally, the interdrop distance does not have similar values under subcritical and supercritical conditions since in the former situation the liquid drops have a motion distinct from that of the surrounding gas whereas in the latter situation, because the densities of the two fluids are more similar, their motion is more similar as well. Moreover, the dense spray effects identified at subcritical conditions were the consequence of both high mass and high volume

loading; in supercritical conditions, the high volume loading is not necessarily accompanied by high mass loading because of the density ratio effect. This means that dense spray effects will be encountered in supercritical conditions only for interdrop distances much smaller than in subcritical conditions. In fact, dense spray effects will visually manifest as small scale density variations, consistent with the discussed observations for coaxial and impinging jets atomization [1, 2, 4, 5].

This study is based upon the isolated fluid drop model developed by Harstad and Bellan [8] and genuinely takes into account fluid drop interaction because the drops are not stationary with respect to each other, but move as a consequence of the  $\text{LO}_x$ – $\text{H}_2$  interdiffusion which changes the radius of the sphere of influence. Thus, the equations developed in [8] for isolated drops are used here with boundary conditions calculated using the global cluster model developed below; in Harstad and Bellan [8] the boundary conditions were prescribed to be those of the surrounding fluid.

## 2. Model

The configuration studied is that of a general-volume-shape cluster of *N* monodisperse fluid drops immersed into a dense gas. The interstitial region between drops is uniform and quiescent with respect to the cluster; the implication is that while turbulence is strong enough to mix the interstitial region, it does not affect the regions near the drops. By definition, the ‘sphere of influence’ around each drop is centered at the drop center and has a radius,  $R_{si}$ , whose value is half of the distance between adjacent drops. The volume of the sphere of influence,  $V_{si}$ , contains the drop and its surrounding fluid, and has a fixed mass; this means that the  $V_{si}$  boundary is a function of time to accommodate the change in density due to heating. The volume of the interstitial region of the cluster is

$$V_i = V_C - NV_{si} \quad (2.1)$$

and it is in this volume that conservation equations are stated below in a Lagrangian frame.

—conservation of mass

$$\frac{d\rho}{dt} = 0 \quad (2.2)$$

—conservation of species *j*

$$\frac{d\rho_j}{dt} = -m_j \nabla \cdot \mathbf{J}_j \quad (2.3)$$

—conservation of energy

$$\frac{d(\rho e)}{dt} = -\nabla \cdot \mathbf{q} \quad (2.4)$$

where  $e = h/m - p/\rho$ .

To obtain the coupling among all fluid drops, we integrate over  $V_i$  and  $S_i$

$$\frac{d}{dt} \left( \int_{V_i} \rho dV \right) = 0 \tag{2.5}$$

$$\frac{d}{dt} \left( \int_{V_i} \rho_j dV \right) = - \int_{S_i} m_j \mathbf{J}_j \cdot d\mathbf{A}_{S_i} \tag{2.6}$$

$$\frac{d}{dt} \left[ \int_{V_i} (\rho h/m - p) dV \right] = - \int_{S_i} \mathbf{q} \cdot d\mathbf{A}_{S_i} \tag{2.7}$$

accounting for the null relative velocity with respect to  $S_i$ .

Equation (2.5) has a simple integral

$$V_i(t) = M_i/\rho_i. \tag{2.8}$$

The species and heat contributions to  $V_i$  arise both from the interaction of the cluster with its surroundings through fluxes of species and heat across  $S_i$ , and from the interaction of  $V_i$  with all drops. Thus, equations (2.6) and (2.7) can be rewritten as

$$\frac{dY_{j,i}}{dt} = \frac{m_j}{M_i} (J_{j,e} A_C - N J_{j,si} A_{si}) \tag{2.9}$$

$$\frac{d}{dt} (h_i/m_i - p_e/\rho_i) = \frac{1}{M_i} (q_e A_C - N q_{si} A_{si}). \tag{2.10}$$

Using thermodynamic relationships, for a two component system, equation (2.10) can be transformed into an equation for  $T_i$

$$\begin{aligned} \tilde{C}_v \frac{dT_i}{dt} = & \frac{1}{M_i} [(q_e - m_1 \tilde{h}_i J_{1,e}) A_C \\ & - N (q_{si} - m_1 \tilde{h}_i J_{1,si}) A_{si}] - \tilde{\kappa}_s \frac{p_e}{\rho_i} \frac{dp_e}{dt} \end{aligned} \tag{2.11}$$

where

$$\tilde{C}_v \equiv C_p/m - \alpha_v \frac{p}{\rho}, \quad \tilde{\kappa}_s \equiv \kappa_s - \tilde{C}_v \frac{m \alpha_v T}{p C_p},$$

$$\tilde{h} \equiv \left( \frac{h_1}{m_1} - \frac{h_2}{m_2} \right) - p \left( \frac{v_1}{m_1} - \frac{v_2}{m_2} \right).$$

The values of  $J_{1,si}$  and  $q_{si}$  are calculated at the edge of the sphere of influence according to the model of Harstad and Bellan [8].

To calculate  $J_{1,e}$  and  $q_e$ , we recall the formalism introduced in [8] where the fluxes were calculated from a transport matrix according to the fluctuation theory of Keizer [9]. In this formalism, the fluxes are linear combinations of the temperature, pressure and mass fraction gradients where the multiplying factors for each term are calculated from transport properties and thermodynamic data such as equations of state. Since in the present study it is only the temperature and the mass fractions that vary in the cluster surroundings, for a binary composition ( $LO_x-H_2$ )

$$J_{1,e} = A_j^{(0)} \frac{\partial Y_1}{\partial x} + B_j^{(0)} \frac{\partial T}{\partial x}, \quad q_e = A_q^{(0)} \frac{\partial T}{\partial x} + C_q^{(0)} \frac{\partial Y_1}{\partial x} \tag{2.12}$$

where  $j = 1$  refers to  $LO_x$ ,  $x$  is the direction normal to the cluster surface enclosed by  $V_i$  and

$$A_j = \frac{m n D_m \alpha_D}{m_1}, \quad B_j = \frac{m_2 n D_m (X_1 X_2 \alpha_T - \overline{\Delta h})}{m T} \tag{2.13}$$

$$A_q = \lambda - k_m \overline{\Delta h}, \quad C_q = \frac{m^2}{m_1 m_2} k_m T \alpha_D \tag{2.14}$$

$$k_m \equiv \alpha_T R_u n D_m, \quad \overline{\Delta h} = \frac{m_1 m_2 X_1 X_2}{m R_u T} \left( \frac{h_1}{m_1} - \frac{h_2}{m_2} \right). \tag{2.15}$$

So far, the formulation is valid for any volume shape. Continuation of the model requires however the choice of a cluster volume shape; since the spherical shape is the simplest in this context, all further results will be valid for this particular case with the understanding that they could be generalized.

For a spherical cluster, the gradients at the cluster interface  $S_C$  can be approximated by a difference across an external length scale  $r_e$  as follows

$$\frac{\partial Y_1}{\partial r} = \frac{Y_{1e} - Y_{1i}}{r_e}, \quad \frac{\partial T}{\partial r} = \frac{T_e - T_i}{r_e} \tag{2.16}$$

where  $r_e$  is related to what is equivalent to a Nusselt number and to the cluster radius through

$$r_e = R_C / Nu_C \tag{2.17}$$

since it is the characteristic distance over which transport occurs at the cluster surface. For a nonspherical cluster, an appropriate expression of  $Nu_C$  and a dimension equivalent to  $R_C$  (for example,  $R_C = [3V_C/(4\pi)]^{1/3}$ ) could be used to define  $r_e$ .

Replacing equations (2.13)–(2.17) into equation (2.12) yields

$$\begin{aligned} & (q_e - m_1 \tilde{h}_i J_{1,e}) A_C \\ & = [3V_C/(4\pi)]^{1/3} 4\pi Nu_C [(A_q^{(0)} - m_1 \tilde{h}_i B_j^{(0)}) (T_e - T_i) \\ & + (C_q^{(0)} - m_1 \tilde{h}_i A_j^{(0)}) (Y_{1e} - Y_{1i})] \end{aligned} \tag{2.18}$$

$$J_{1,e} A_C = [3V_C/(4\pi)]^{1/3} 4\pi Nu_C [A_j^{(0)} (Y_{1e} - Y_{1i}) + B_j^{(0)} (T_e - T_i)] \tag{2.19}$$

which provides the necessary information for equations (2.9) and (2.11). In this formulation,  $Nu_C$  is a parameter whose value is prescribed.

### 3. Solution procedure

The solution of the drop in its sphere of influence is discussed in detail in Harstad and Bellan [8]. In the interstitial region, the unknowns are  $T_i$  and  $Y_{1i}$  since  $\rho_i$  is determined by the state equation. Once  $\rho_i$  is known,  $V_i$  and  $V_C$  are calculated. During each time step, the total

differential equations (2.9) and (2.11) are solved using a second order predictor–corrector method in the manner described in [8].

#### 4. Results

As discussed above, due to the essentially diffusive behavior at high pressure it is expected that dense spray effects will be important only for fluid-drops in closer proximity than in subcritical conditions. An experimentally verified [10] classification of various subcritical spray regimes according to the magnitude of the ratio  $R_{si}/R_d$  identified the very dense regime as that corresponding to  $R_{si}/R_d \leq 10$  [11]. Calculations with the above model were performed with a relatively large value of  $Nu_C$ ,  $Nu_C = 100$ , in order to simulate the enhanced heat transfer for fluids at supercritical conditions [12]. The results obtained with  $R_d^0 = 50 \times 10^{-4}$  cm,  $R_{si}^0 = 8R_d^0$ ,  $R_C^0 = 2$  cm,  $T_{db}^0 = 100$  K ( $T_d^0$  is uniform inside the drop),  $T_{si}^0 = T_e = 1000$  K,  $p_e = 20$  MPa and  $Y_{1e} = 0$  were essentially identical to those of the isolated drop exposed to surroundings at the external cluster conditions [8]. This finding indicates that under the conditions of this study, dense spray effects will indeed be encountered at much smaller values of  $R_{si}/R_d$ .

The results presented below parallel that of Harstad and Bellan [8] for isolated drops. First, a baseline behavior is identified for dense spray effects and compared with the findings of [8] for external cluster conditions identical to the surrounding conditions of the isolated drop. Then, a parametric study follows paralleling that of [8] and the parametric variations found here are compared to those found in that study.

##### 4.1. Baseline behavior and effect of the initial radius of the sphere of influence

The typical situation considered is:  $R_d^0 = 50 \times 10^{-4}$  cm,  $R_{si}^0 = 2R_d^0$ ,  $R_C^0 = 2$  cm,  $Nu_C = 100$ ,  $T_{db}^0 = 100$  K ( $T_d^0$  is uniform inside the drop),  $T_{si}^0 = T_e = 1000$  K,  $p_e = 20$  MPa and  $Y_{1e} = 0$ . The number of drops in the cluster is  $5.92 \times 10^6$ .

Figure 1 illustrates the spatial profiles of  $T$ ,  $Y_1$  and  $\rho$  at different times while Fig. 2 depicts the time evolution of  $T_{si}$ ,  $Y_{1,si}$ ,  $\rho_{si}$ ,  $N_d$ ,  $R_{si}$  and  $R_C$ . Comparisons between the calculated baseline results and those obtained for  $R_{si}^0 = 5R_d^0$  and  $R_{si}^0 = 10R_d^0$  (considered here typical of no drop interactions [8]) under otherwise identical conditions are presented in Fig. 3. While the baseline spatial variation of  $T$ ,  $Y_1$  and  $\rho$  parallels that obtained for the isolated drop in that it is mainly a diffusion process, the closer drop proximity induces smaller density gradients (Fig. 3(c)), steeper  $Y_1$  gradients (Fig. 3(b)) and larger temperatures as the initial interstitial temperature is equal to that of the cluster surroundings. An important differ-

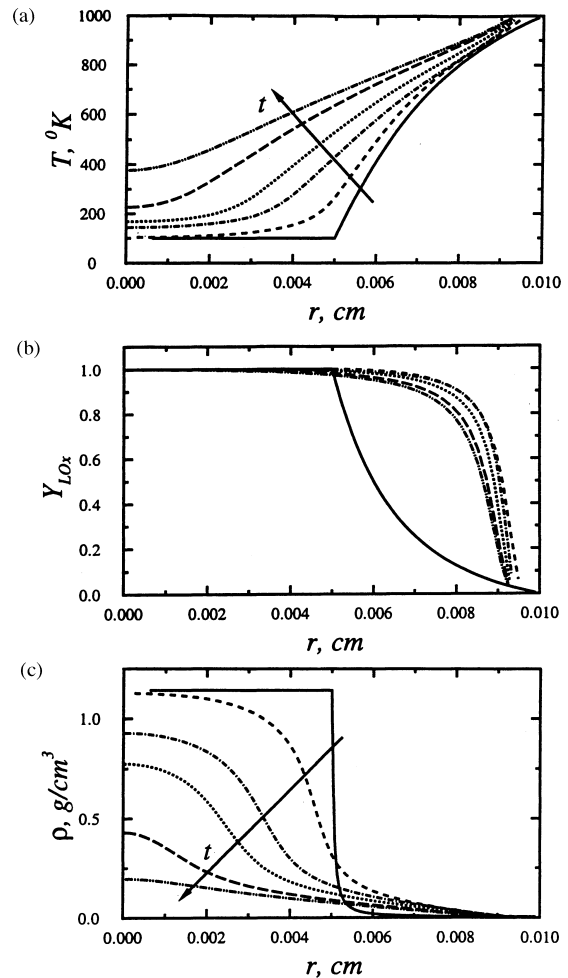


Fig. 1. Spatial variation of the temperature (a), oxygen mass fraction (b) and density (c) at various times for  $R_d^0 = 50 \times 10^{-4}$  cm,  $R_{si}^0 = 2R_d^0$ ,  $R_C^0 = 2$  cm,  $T_{db}^0 = 100$  K,  $Nu_C = 10^2$ ,  $T_{si}^0 = T_e = 1000$  K,  $p_e = 20$  MPa and  $Y_{1e} = 0$ . The curves correspond to the following times: 0.0 s (—);  $2 \times 10^{-3}$  s (---);  $6 \times 10^{-3}$  s (-·-·-);  $8 \times 10^{-3}$  s (···);  $1 \times 10^{-2}$  s (- - -);  $1.08 \times 10^{-2}$  s (-·-·-).

ence among the results obtained with decreasing  $R_{si}^0/R_d^0$  is the increased accumulation of a non-negligible amount of  $LO_x$  in the interstitial region (see Fig. 2(a)) as the cluster contracts due to heat being transferred faster from the interstitial region to the fluid  $LO_x$  drops than it is replenished from the cluster surroundings. This is consistent with the small decrease of  $T_{si}$ ,  $R_{si}$  and  $R_C$  and the equivalent increase in  $\rho_{si}$  shown in Fig. 2. Eventually,  $T_{si}$  increases due to the increased amount of heat transferred to the cluster from its surroundings, and  $Y_{1,si}$  accordingly decreases. All these effects are small for the values used in this baseline calculation; in particular, the value of  $Nu_C$  is too small for heat transfer from the cluster sur-

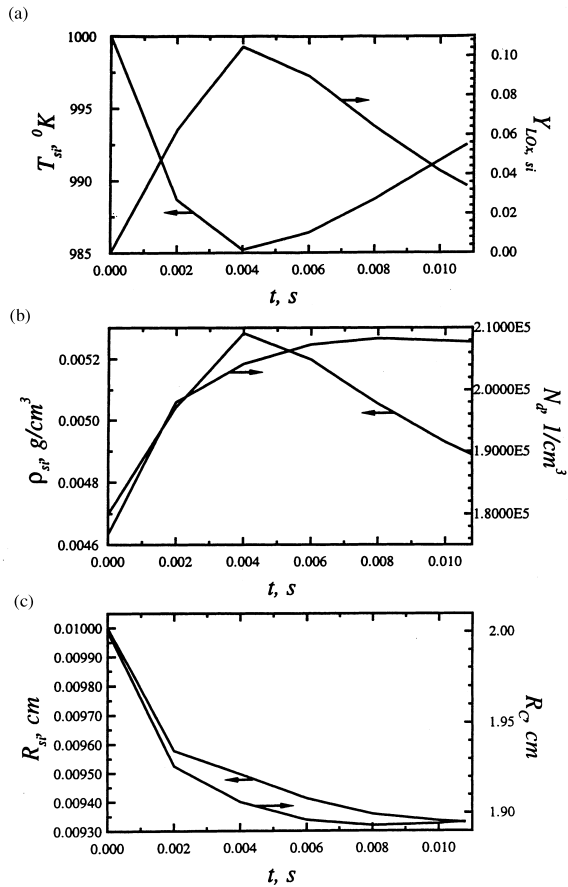


Fig. 2. Time evolution of  $T_{si}$  and  $Y_{1,si}$  (a),  $\rho_{si}$  and  $N_d$  (b), and  $R_{si}$  and  $R_C$  (c) for the same initial conditions as those in Fig. 1 caption.

roundings to replenish the heat relinquished by the interstitial fluid to the fluid drops. In fact, just as in subcritical cluster studies [11], the motion of the cluster boundary is governed by the ratio of two characteristic times: that of heat transferred from the interstitial region to the drops (a heat sink),  $t_1$ , and that transferred from the cluster surroundings to the cluster (a heat source),  $t_2$ ; the only difference with the subcritical studies is the absence of phase change which requires a substantial heat budget. The latent heat carried away by the evaporated compound in subcritical situations constitutes a major percentage of the heat input to the gas phase. This explains the present, relatively undramatic cluster contraction and expansion.

The effect of drop proximity decreases with increasing pressure (Fig. 3 and the more extensive study below), as the behavior becomes increasingly similar to pure diffusion. Additionally, the importance of drop interactions increases with increasing  $t_2$  (decreasing  $Nu_C$ ) since the heat transferred to the drops is replenished more

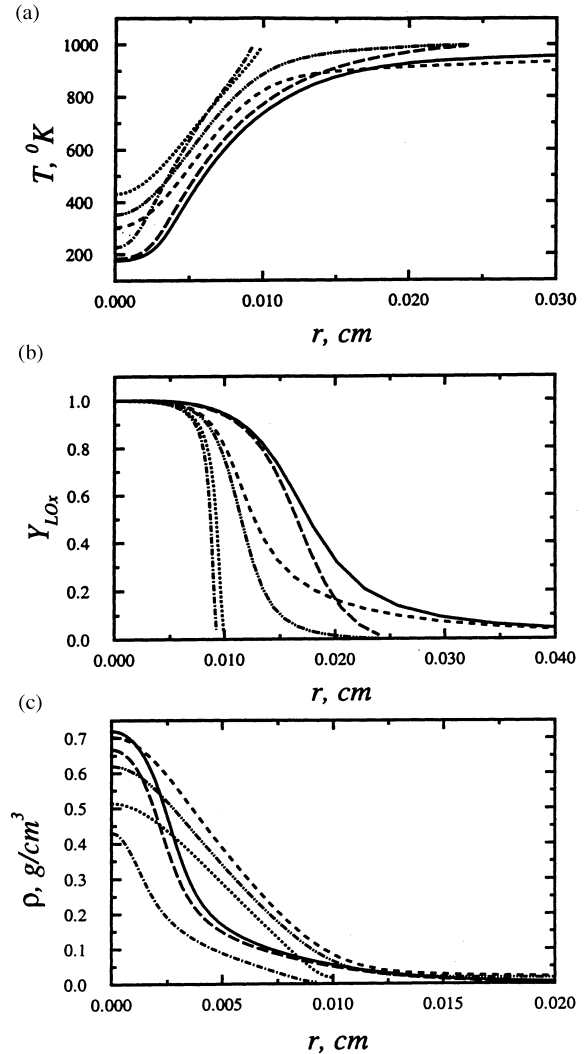


Fig. 3. Spatial variation of the temperature (a), oxygen mass fraction (b) and density (c) at:  $t = 10^{-2}$  s for  $R_{si}^0 = 10R_d^0$  [ $p_e = 20$  MPa (—);  $p_e = 80$  MPa (---)];  $R_{si}^0 = 5R_d^0$  [ $p_e = 20$  MPa (- - -);  $p_e = 80$  MPa (- · - ·)] and  $R_{si}^0 = 2R_d^0$  [ $p_e = 20$  MPa (- · - · -);  $p_e = 80$  MPa (· · ·)]. All other initial conditions are those of Fig. 1 caption.

slowly, and decreasing  $t_1$  [(obtained by varying  $R_d^0$  (see below) or by varying  $R_{si}^0/R_d^0$  (see above)].

Elimination of the spatial variable between  $T$  and  $Y_1$  (Fig. 4) obtained in the baseline case shows that the solution is self-similar with respect to time away from the initial condition; a result difficult to predict a priori.

#### 4.2. Effect of the cluster Nusselt number

As discussed above, the baseline value of  $Nu_C = 10^2$  corresponds to  $t_2 \gg t_1$ . To study true competition

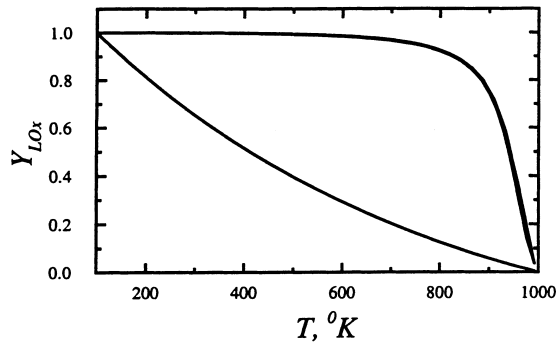


Fig. 4. Oxygen mass fraction vs. temperature for the initial conditions in the Fig. 1 caption.

between these two characteristic times,  $t_2$  was decreased by increasing  $Nu_C$  to, respectively,  $10^3$ ,  $10^4$  and  $10^5$ . The results illustrated in Fig. 5 show a surprisingly weak influence of  $Nu_C$  over a variation spanning three orders of

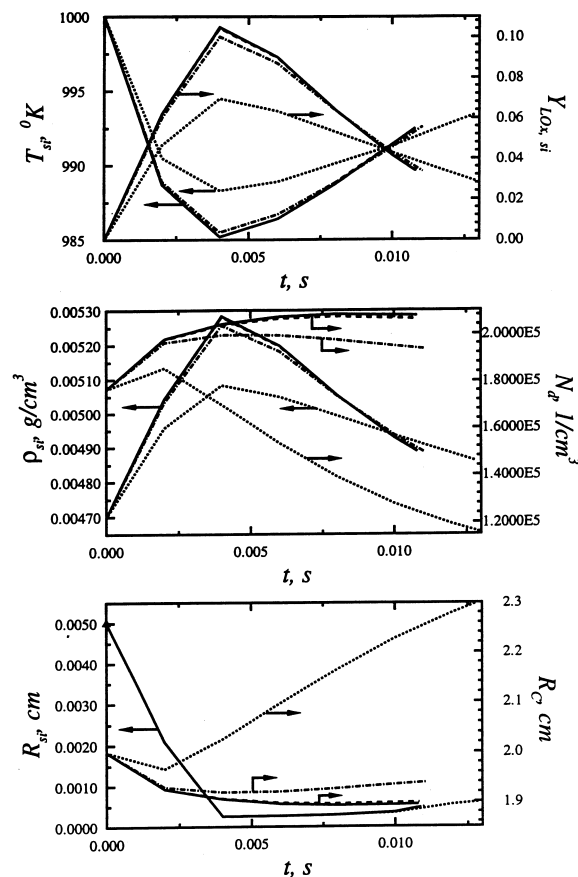


Fig. 5. Time evolution of  $T_{si}$  and  $Y_{1,si}$  (a),  $\rho_{si}$  and  $N_d$  (b), and  $R_{si}$  and  $R_C$  (c) for  $Nu_C = 10^2$ : (—);  $10^3$  (---);  $10^4$  (-·-·-); and  $10^5$  (···). The other initial conditions are those of Fig. 1 caption.

magnitude. In agreement with the physics, with increased  $Nu_C$  the heat flux to the cluster increases and results in larger  $T_{si}$ , increased cluster expansion and therefore smaller  $Y_{1,si}$  and  $\rho_{si}$ .

The modeling of heat transfer to the cluster through the  $Nu_C$  is somewhat uncertain under supercritical conditions since the approach assumes the same length scale for  $T$  and  $Y_1$ . This assumption becomes increasingly questionable with increasing pressure and lack of experimental data precludes any definitive model.

#### 4.3. Effect of the initial cluster radius

Since heat transfer to the cluster increases with the ratio  $S_C/V_C$  and thus with decreasing  $R_C^0$ , calculations were performed with baseline initial conditions except that now  $R_C^0 = 1$  cm. Temporal plots (not presented) of  $T_{si}$ ,  $Y_{1,si}$ ,  $\rho_{si}$ ,  $N_d$ ,  $R_{si}$  and  $R_C$  show that the results are virtually identical to those obtained with the larger cluster. Consistent with the weak  $Nu_C$  effect, the decrease in the number of drops in the cluster by a factor of eight is inconsequential since the drop behavior is not governed by heat transfer to the cluster.

#### 4.4. Effect of the initial fluid drop radius

The previous changes in the values of  $Nu_C$  and  $R_C^0$  resulted in the variation of  $t_2$ ; however, the ratio  $t_1/t_2$  can also be changed by manipulating  $t_1$ . This is accomplished here by increasing  $R_d^0$  to  $100 \times 10^{-4}$  cm. Since  $R_{si}^0 = 2R_d^0$  and the cluster surface is the outer envelope of all spheres of influence, the number of drops decreases to  $7.4 \times 10^5$ .

Comparisons between the results obtained with larger drops and the baseline results appear in Fig. 6 for three different pressures. The larger drops heat up more slowly, maintain steeper density gradients for extended times and exhibit a larger variation in  $Y_1$  with increasing pressure. In fact, plots of the interstitial values of  $T_{si}$ ,  $Y_{1,si}$  and  $\rho_{si}$  show that the same values are reached, but at larger times; thus, the effect of larger drops is that of a change in the characteristic time of the diffusion process that dominates at supercritical conditions.

#### 4.5. Effect of the surrounding pressure

To investigate the impact of drop interactions with surrounding pressure, extensive calculations were performed by changing  $p_c$  by almost an order of magnitude from 10–80 MPa. The results are depicted in Fig. 7.

Unlike in the isolated drop situation [8] where gradients were greater with increasing pressure, here it is exactly the opposite. The effect of the drop interaction is to smear the gradients by increasing the cluster volume with increasing pressure. This volume increase is a direct consequence of the reduction in  $\rho C_p$  with increasing

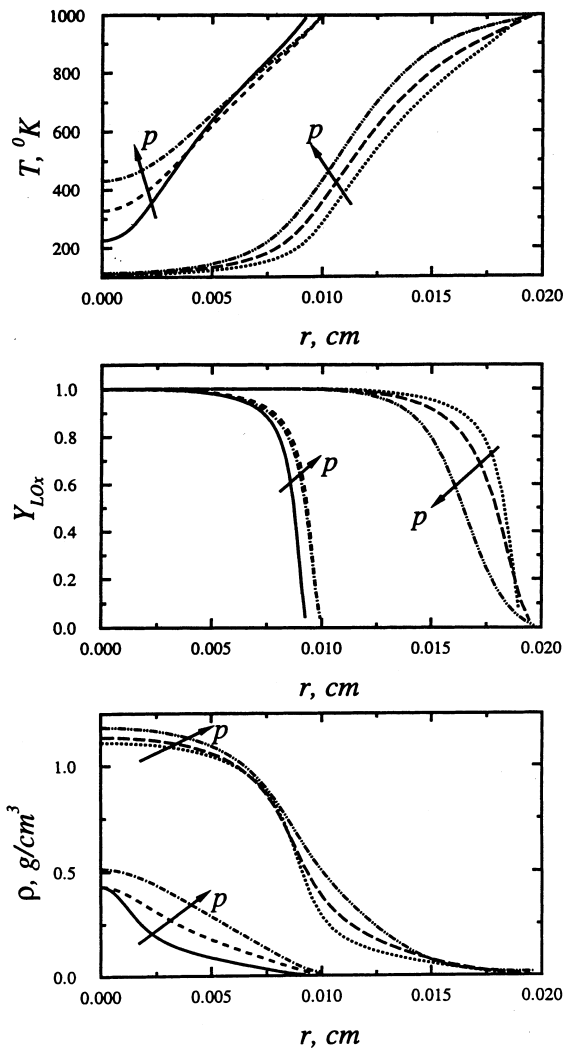


Fig. 6. Spatial variation of the temperature (a), oxygen mass fraction (b) and density (c) at:  $t = 10^{-2}$  s for  $R_d^0 = 50 \times 10^{-4}$  cm [ $p_e = 20$  MPa (—);  $p_e = 40$  MPa (---);  $p_e = 80$  MPa (-·-·-)] and  $R_d^0 = 100 \times 10^{-4}$  cm [ $p_e = 20$  MPa (·-·-);  $p_e = 40$  MPa (-·-);  $p_e = 80$  MPa (-·-·-)]. The other initial conditions are those of Fig. 1 caption.

pressure in the predominantly- $\text{LO}_x$  side of the interface; this results in a temperature augmentation in this region, which increases the average cluster temperature, and thus, its volume. A more minor effect, which increases the available heat source, is the slight elevation of the interstitial molar enthalpy with increasing pressure. Thus, the presence of clusters of drops in burning sprays tends to render the dependent variables more uniform with increasing pressure whereas the opposite is true for isolated drops. In this respect, clusters of drops are a desir-

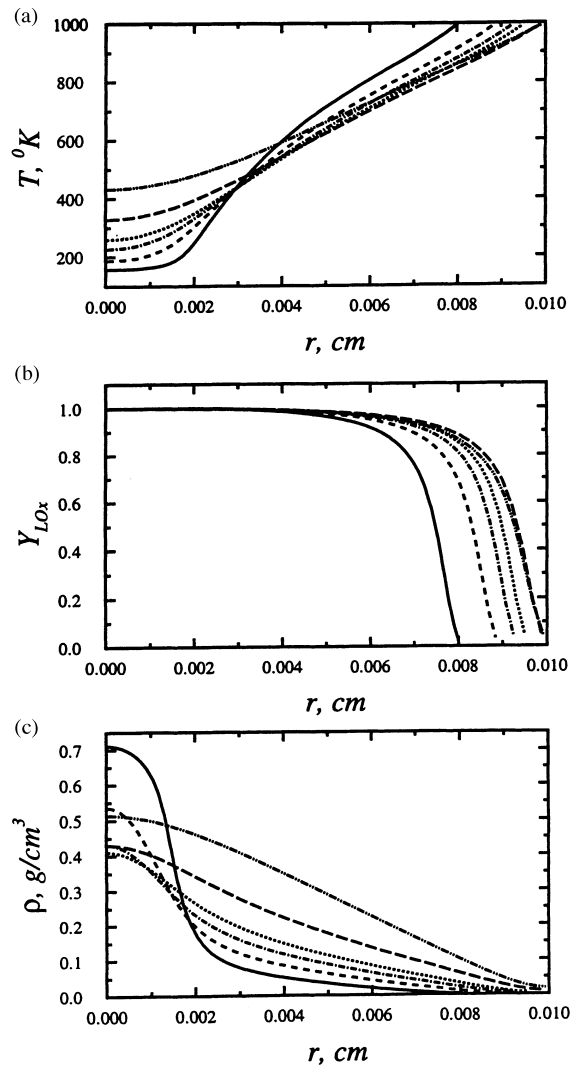


Fig. 7. Spatial variation of the temperature (a), oxygen mass fraction (b) and density (c) at  $1 \times 10^{-2}$  s for several pressures: 10 MPa (—); 15 MPa (---); 20 MPa (-·-·-); 25 MPa (·-·-); 40 MPa (-·-); 80 MPa (-·-·-). Other initial conditions are those of Fig. 1 caption.

able aspect because they aid the interdiffusion of the reactive components.

### 5. Summary and conclusions

A model of fluid drop behavior in clusters has been developed based upon a previous formulation for an isolated fluid drop. Transfer of heat and mass to the cluster from its surroundings has been modeled using a Nusselt number approach. Results obtained by varying

the Nusselt number over three orders of magnitude show that the cluster behavior is not sensitive to the value of the Nusselt number. Consistent with this finding, the size of the cluster is also found to be a very weak parameter.

Since atomization of  $LO_x$  sprays into small fluid drops is considered desirable to design engineers, we have also studied the impact of the fluid drop size on the mass fraction distribution within the cluster. Larger fluid drops have a larger characteristic time and evolve similarly to the smaller drops except that the evolution time is proportionally stretched. Our results, therefore, agree with the well-known design practice of trying to atomize the  $LO_x$  jet into the smallest possible size fluid drops.

Parametric studies performed for the  $LO_x$ - $H_2$  system for various drop proximities show that the most important cluster effect is the accumulation of a non-negligible amount of  $LO_x$  with decreasing drop interdistance. The effect of drop proximity decreases with increasing pressure in that the behavior of the fluid drops in a very dense gas becomes increasingly similar to a pure diffusion process. For given initial drop proximity, an increase in pressure results in increased smearing of the gradients, a desirable aspect because it promotes interdiffusion. This result is exactly the opposite of what is obtained for isolated fluid drops [8].

The effect of decreasing the surrounding cluster temperature, although not discussed here, is to reduce the available heat source for the fluid drops, and thus to decrease the rate of cluster expansion, as well as to reduce the relaxation rate of the fluid drop density.

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